AD-A260 958

TATION PAGE

Form Approved
OMB No 0704-0188

(2)

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED FINAL/01 OCT 89 TO 30 SEP 92
4. TITLE AND SUBTITLE DISCRETE METHODS AND THEIR APPLICATIONS		5. FUNDING NUMBERS 2304/A8 AFOSR-90-0008
6. AUTHOR(S) DR. GUNZBURGER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) THE STATE UNVIERSITY OF NEW JERSERY RUTGERS P.O. BOX 1089 PISCATAWAY NJ 08854-1089		8. PERFORMING ORGANIZATION REPORT NUMBER AFOSR-TR- 93 0143
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR/NM 110 DUNCAN AVE, SUTE B115 BOLLING AFB DC 20332-0001		10. SPONSORING/MONITORING AGENCY REPORT NUMBER AFOSR-90-0008
11. SUPPLEMENTARY NOTES		
12a. DISTRIBUTION/AVAILABILITY STATEMENT APPROVED FOR PUBLIC RELEASE: DISTRIBUTION IS UNLIMITED		12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

A new algorithm for clustering which contructs better cluster than the well known k-means algorithms has been developed. A new column generation approach has allowed the solution of very large tool-generation problems.

14. SUBJECT TERMS			15. NUMBER OF PAGES
			16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT SAR(SAME AS REPORT)

FEB 23 1993

RUTCOR

FINAL TECHNICAL REPORT

To Air Force Office of Scientific Research

Discrete Methods and their Applications

Grant Number AFOSR 90-0008

Peter L. Hammer
Co-Principal Investigator

Fred S. Roberts
Co-Principal Investigator

February 3, 1993

RUTCOR

Discrete Methods and their Applications

Grant Number AFOSR 90-0008

FINAL TECHNICAL REPORT

With an Emphasis on Research Accomplishments: October 1, 1991 - September 30, 1992

This summary of research accomplishments is organized into essentially the same sections and subsections as is our original proposal. It emphasizes research accomplishments in the last year of the project, October 1, 1991 to September 30, 1992. The reader is referred to the annual technical reports for research accomplishments of earlier years. Papers referred to by number are listed below in the list of publications prepared under the grant during the period October 1, 1991 to September 30, 1992. Papers referred to with authors' names and year are listed at the end of the section.

1. Discrete Methods Closely Tied to Applications

Applications play a central role in this project and in the history of discrete mathematics. Several of the discrete methods we have studied are very explicitly tied to specific applications. These have been a fundamental part of our research. They are described in some detail in this section.

1.1. Location Problems

Location problems arise whenever a large set of potential sites for placing certain units is available and a selection must be made of the sites to be utilized. Such problems arise naturally in situations like placing warehouses, satellites, communication centers, military units, or emergency services. They are especially important in Air Force problems involving locations of hubs, points of embarkation, staging areas, and other facilities; such problems arise frequently in the network routing questions at MAC or the Air Force Logistics Center. See Hansen, et al. [1987] for a recent survey. We have been studying a variety of location problems and approaches to solving them.

Weber's problem, which consists of locating a single facility in order to minimize the sum of Euclidean distances between its location and those of a given set of users, is the most studied one of continuous location theory. Since its statement in Weber [1909], many generalizations have been considered. One of these is the ***multifacility Weber problem*** where several facilities are to be simultaneously located in the Euclidean plane in order to minimize the sum of weighted distances between a given set of users and their closest facilities. A second is the ***conditional Weber problem*** which

arises when some facilities are already established, and the problem is to locate a new facility in order to reduce the most the sum of distances from the users to the closest existing or new facility. Paper [13] and thesis [11] are concerned with applications of d.-c. programming to these generalizations of the Weber problem. ***D.-c. programming*** is a recent technique of global optimization which allows the solution of problems whose objective function and constraints can be expressed as differences of convex functions. We have obtained good computational results for problems with up to a thousand users, twenty existing facilities, and three new facilities.

Paper [14] and thesis [11] are also concerned with aspects of Weber's problem. These publications extend work of last year which was concerned with the case where some weights are positive and some are negative. This case was also shown to be a d.-c. program, reducible to a problem of concave minimization over a convex set. The latter is solved by outer-approximation and vertex enumeration. Moreover, locational constraints can be taken into account by combining the previous algorithm with an enumerative procedure on the set of feasible regions. The major new result is that the algorithm has been extended to solve the case where the obnoxiousness of the facility is assumed to have exponential decay. The paper reports on computational experience with up to 1000 users.

Traditionally in the theory of location problems, one is interested in minimizing or maximizing some objective function and this objective function is assumed a priori. Since there are so many potential objective functions, attention must be paid to how to choose an appropriate function. Recently, Holzman [1990] specified some reasonable conditions that an objective function for solving a location problem should satisfy. He showed that as long as the network in which the facilities were to be located had a tree structure, then the objective function was uniquely determined by these conditions. Vohra [1990] obtained similar results for trees under other conditions. In paper [45], we have found a simpler result than Vohra's which also holds in a variety of different contexts, in particular which allows both the users and the facilities to either only be at vertices of the network or to be anywhere along the edges.

Networks with tree structures are rather special and so the results of Holzman, Vohra, and paper [45] referred to above are somewhat special. We have investigated more general networks. In earlier years, we had found that a reasonable set of axioms related to Holzman's is self-contradictory for some non-tree networks. In the past year, we have found the startling result that under very general conditions closely related to Holzman's, for all connected networks other than trees, there is no objective function satisfying these conditions. By omitting one of the assumed conditions, one can restate the result in the following way: Under some reasonable conditions which one would like to impose on the objective function in solving a location problem, the solution can be very sensitive to small changes in locations of the users of the facilities. This has important practical implications for implementing the solutions to

real location problems. The results are written up in paper [29].

1.2. Clustering Problems

In many practical problems of detection, decisionmaking, or pattern recognition, we seek methods for clustering alternatives into groups. Clustering methods aim at finding, within a given set of entities, subsets called clusters which are both homogeneous and well-separated. Clustering methods have been used for solving a variety of problems of interest to the Air Force. For instance, they have been used at MAC in locating (through the OADS model) U.S. hubs at Travis Air Force Base in California and Tinker Air Force Base in Oklahoma; in identifying good points of embarkation in deliberate planning models; in identifying staging areas for medical evacuations; and in identifying hubs for the defense courier system. Clustering methods are also relevant to the analysis of various practical problems of the Air Force which involve large amounts of data. These problems arise in such diverse contexts as early warning systems, detection of enemy positions, remote operations in space, cargo movement, "troubleshooting" in complex electronic systems, and forecasting.

A well-known clustering algorithm, the ***k*-Means algorithm**, partitions elements (in Euclidean space) into two distinct clusters according to a dissimilarity measure (the Euclidean distance). The algorithm is based on the minimization of a certain functional, through a descent procedure. Although this algorithm runs quite fast, quickly detecting clusters in large data sets, it misclassifies a great number of objects when clusters are quite different in size. We have devised a new algorithm, based on quadratic 0-1 minimization. This algorithm constructs a much better classification; in tests, it has repeatedly almost completely retrieved the original clusters which were randomly generated selecting objects from two normal distributions. See paper [5].

A quite different application of clustering arises in problems of planning and scheduling in automated manufacturing. Automated manufacturing systems play a vital role in increasing productivity in manufacturing. The most recent among these systems are characterized by extremely high levels of speed and accuracy. Unfortunately, the complexity of the planning problems associated with these systems usually increases in direct relation to their technological sophistication. We have made important progress on several planning and scheduling problems related to automated manufacturing. Specifically, we have successfully addressed several types of tool management problems. A central issue in tool management for flexible manufacturing systems consists in deciding how to sequence the parts to be produced and what tools to allocate to the machines in order to minimize the number of tool setups (which disrupt the production flow). We have developed a "column generation" approach which has allowed us to solve to optimality much larger instances of certain tool-generating problems than those previously handled in the literature. The results are written up in paper [15].

We have been working on a number of problems that derive clustering from judgements of closeness. In particular, we have studied the interval graph model in which we start with judgements of closeness, assign to each element being judged a real interval, and take two intervals to overlap if and only if the corresponding elements are judged close. This can be accomplished if and only if the graph whose vertices are the elements and whose edges correspond to closeness defines an *interval graph*. Interval graphs arise in numerous applications, including problems involving scheduling, transportation and communications, computer systems, ecosystems, foundations of computation, genetics, and seriation in archaeology and psychology. Specifically, we have studied interval graphs in connection with no-hole T-colorings; see Section 2.1. We have also studied *unit interval graphs*, interval graphs where all of the real intervals have the same length, and *$n^{\sim}$-graphs*, where the real intervals are replaced by sets of n consecutive integers. The $n^{\sim}$-graphs arise in problems of visual perception, involve perceptual judgements of betweenness and proximity that might be relevant to those made by pilots or radar systems, and go back to the work of Goodman [1951] on perceptual geometry. Roberts [1979] showed that the class of unit interval graphs and the union of the classes of $n^{\sim}$-graphs are the same. We have studied the problem of finding the minimum n such that a given unit interval graph is an $n^{\sim}$-graph. A linear time algorithm to compute this number in a particular case is given, improving the earlier algorithms by Fine and Harop. An (integer) linear programming formulation has also been obtained. These results are discussed in the paper [51] and the thesis [50].

1.3. Global Optimization

In operations research, one makes the distinction between algorithms designed to find a local optimum and algorithms designed to find the global optimum. The vast majority of nonlinear programming algorithms belong to the first category, but increasing attention is being devoted to the latter one. We have found that many of the ideas underlying algorithms for combinatorial optimization can be transposed to the field of global optimization and we have been exploring this idea.

In Section 1.1, we have described our work on d.-c. programming and its applications to Weber's problem of location theory and various generalizations of that problem. As we remarked there, d.-c. programming is a recent technique of global optimization that allows the solution of problems whose objective function and constraints can be expressed as differences of convex functions. These results are described in papers [13] and [14] and thesis [11].

Building on work begun in the previous year, we have been concerned with reduction of global optimization problems to bilinear programs. A *bilinear program* involves minimization of a bilinear function subject to bilinear constraints. In paper [27], we concentrate on the special case of quadratic programs. General quadratic programs appear to be very difficult to solve exactly. However, reduction of such programs to bilinear programs by

duplication of variables appears to be a fruitful approach. We have studied the problems of finding such reductions in which either the number of additional variables is minimum or the number of complicating variables, i.e., variables to be fixed in order to obtain a linear program in the resulting bilinear program, is minimum. In paper [27], begun last year, these two problems are shown to be equivalent to a maximum bipartite subgraph and a maximum stable set problem respectively in a graph associated with the quadratic program. Non-polynomial but practically efficient algorithms for both reductions are thus obtained. Reduction of more general global optimization problems than quadratic programs to bilinear programs is also discussed.

1.4. Applications of Discrete Mathematics to Decisionmaking

Problems involving complex choices are often most naturally formulated using discrete mathematics. The tools of discrete mathematics are widely used in the literature of individual decisionmaking, group decisionmaking, game theory, and measurement and utility theory. We have been using discrete techniques to investigate problems involving decisionmaking and concepts from the theory of decisionmaking to attack other problems of discrete mathematics.

In many situations, multiple decisionmakers with divergent objectives intervene in decisions to be made. The simplest such case, in which there are only two decisionmakers, has long been studied in game theory. If there is some asymmetry between the decisionmakers, in that one of them, called the *leader*, makes his decisions first, anticipating the reaction of the other one, called the *follower*, and cooperation is ruled out a priori, we have what is called a *Stackelberg game*. Adding joint constraints on the strategies of the leader and the follower makes the model more realistic and leads to *bilevel programming*, a topic which has attracted much attention recently. Building on work begun in earlier years, we have studied linear bilevel programming (see paper [28]). We have proposed a new branch-and-bound algorithm for linear bilevel programming. We have used necessary optimality conditions expressed in terms of tightness of the follower's constraints to fathom or simplify subproblems, branch and obtain penalties similar to those used in mixed-integer programming. Our computational results compare favorably to those of previous methods and we have been able to solve problems with up to 150 constraints, 250 variables controlled by the leader, and 150 variables controlled by the follower.

Most military decisionmaking takes place in a climate of uncertainty and risk. Theories of risk have a long history in the literature of decision theory. Continuing work begun in the previous year, we have presented in paper [17] a discussion of the prediction in the economic theory of choice that, under appropriate restrictions on the shape of the objective function, all risk-averters reduce their activities when they are shifted from a certainty environment to a risky one with the same mean. When mean preserving 'marginal' changes in risk - instead of "global" ones -

are considered, risk aversion is no longer sufficient to predict the decisionmaker's response even for simple problems with a linear objective function. Hence, further restrictions have to be imposed in order to find again the kind of result obtained in the "no risk to risk" case. The search for well-behaved results has led economists to pay attention almost exclusively to the utility function. We have made considerable progress by considering the other ingredient of the decision problem, the distribution function of the random variable. We build on earlier work of Meyer and Ormiston and others by extending the class of changes in the distribution that give unambiguous comparative statics results. We do so by comparing distributions with stochastic dominance in the tails.

In recent years, a considerable amount of discrete mathematical research has been brought to bear on methods for understanding and improving group decisionmaking. Two of the most useful group decisionmaking procedures are the median procedure and the closely related mean procedure. These procedures are used in aggregating preferences in social choice problems; in voting; in operations research as solutions to location problems; in statistical analysis as concepts of centrality; in molecular biology in the process of finding consensus patterns in molecular sequences; and in mathematical taxonomy in finding the consensus in classification problems. In paper [49] we present a general framework for speaking about the median and mean procedures and use it to organize a wide variety of recent results. In particular, we are able to show how various approaches to finding the objective function in location problems on networks (see the discussion in Section 1.1) fall into this framework.

The thesis [1] is concerned with the stable matching problem. In this problem, agents must be matched in pairs while having preferences over their potential mates. The goal is to find a matching where no matchable pair of agents prefer each other to their outcome under the matching. Such a *matching* is called *stable*. Stable matchings have a variety of practical uses, in particular in assignment of residents to hospitals. Stable matchings are a generalization of the *stable marriage problem*, the special case where each agent is labelled as either a man or woman and each matchable pair consists of a man and a woman. Gale and Shapley showed that a stable marriage problem always has a stable matching. We show that their algorithm can be interpreted as a dual simplex method. We characterize the graphs for which all associated stable matching problems have stable matchings. Irving obtained the first polynomial algorithm for non-bipartite problems that finds a stable matching or determines that none exists. We show that linear programming, combined with a simple graph search procedure, yields an alternative polynomial algorithm.

1.5. Multiple Conclusion Logic

When a given signal can be interpreted as being the result of a variety of causes and a small number of tests have to be created to identify the exact cause of the signal, we have a typical instance

of a multiple conclusion logic situation. Examples of such situations occur in medical decisionmaking, in "troubleshooting" in complex systems including networks and electronic and mechanical systems, in searching and seeking in hazardous or nuclear or chemically toxic environments, in detecting enemy positions, in remote operations in space or underseas, and so on. We have worked on a number of approaches to such problems.

Specifically, we have studied the multiple conclusion logic problem involving involving optimal compression of knowledge bases in expert systems. In papers [24] and [26], this is formalized as the problem of boolean function minimization and is studied using the concept of Horn function. Further work on Horn functions and Horn formulae and related work on the satisfiability problem is contained in our papers [25], [7], and [9] and the thesis [52] and is discussed in detail in Section 3.1.

2. Discrete Structures and their Applications

Discrete structures such as graphs, partially ordered sets, block designs, and matroids have a wide variety of applications in practical problems. We have investigated a variety of such structures and their applications in this project, with an emphasis on graphs.

2.1. Graph Coloring, Stability, and their Applications

Our research in graph theory has been closely tied to applications. The applications we have considered involve primarily questions of communications and transportation and basic problems in operations research such as scheduling, maintenance, and assignment problems. The specific mathematical questions we have investigated in graph theory are divided into three areas, in this and the next two subsections.

Much current work in graph theory is concerned with the related problems of finding optimal graph colorings and finding the largest stable set in a graph. Problems of graph coloring and stability have been a major area of emphasis for us.

Graph colorings have a wide variety of applications in scheduling, fleet maintenance, traffic phasing, etc. (For a discussion of the applications of graph coloring, see Roberts [1984,1991a].) We have been working on some fundamental problems of graph coloring. In particular, we have studied ***list colorings*** of graphs. In many practical coloring problems, a choice of color to assign is restricted. A set or list of possible colors to be assigned to a vertex is specified, and we seek a graph coloring so that the color assigned to a vertex is chosen from its list. List colorings arise for instance in channel assignment problems when we specify possible acceptable channels. We have also studied the variant on list colorings that arises when we seek a graph coloring so that the color assigned to a given vertex must not belong to a set of disallowed colors. Erdős, Rubin and Taylor [1979] introduced the idea of considering when a graph G can be list-colored for

every assignment of lists of k colors in each list. If G can always be list colored for every such assignment, we say that G is ***k-choosable***. Following Brown, et al. [1990], we say that G is ***(j,m)-amenable*** if whenever subsets $R(x)$ of $\{1,2, \ldots,m\}$ of size j are assigned to vertices x of G, there is an ordinary graph coloring using colors in $\{1,2,\ldots,m\}$ so that the color assigned to x does not belong to set $R(x)$. Motivated by the relationship between k-choosability and (j,m)-amenability, we say that graph G satisfies property $P(k,J)$ with $0 \leq J \leq \infty$ if for all $j > 0$, G is $(j,j+k)$-amenable if and only if $j \leq J$. In paper [44], we start with the simple observation that G is k-choosable if and only if it is k-colorable and has property $P(k,\infty)$ and we study the problem of identifying graphs having properties $P(k,J)$.

In related work, in paper [31], we solve a problem posed by Erdös, Rubin, and Taylor [1979] by finding an asymptotic result for the list chromatic number (the smallest k so that a graph is k-choosable) of the random graph $G(n,1/2)$.

We have been studying T-colorings of graphs in connection with frequency assignment problems. In such problems, the vertices of a graph G represent transmitters and an edge between two vertices represents interference. We seek to assign to each vertex or transmitter x a channel $f(x)$ over which x can transmit, and for simplicity we take the channels to be positive integers. The assignment of channels is subject to the restriction that if two transmitters interfere, then the channels assigned to these transmitters cannot be separated by a disallowed distance. To make this more precise, we fix a set T of nonnegative integers and assign channels so that if vertices x and y are joined by an edge of G, then $|f(x)-f(y)|$ is not in T. The assignment f is called a ***T-coloring***. Its ***span*** is $\max |f(x)-f(y)|$ over all pairs of vertices x and y. See Roberts [1991b] and Tesman [1989] for a summary of the literature of T-colorings and a statement of some of the fundamental problems. In the thesis [50], we have extended and compiled work in earlier years on ***no-hole T-colorings***, T-colorings in which $\cup\{f(x): x \in V(G)\}$ is a set of consecutive integers. Heuristic algorithms for T-colorings developed at NATO by T. Lanfear are based on the idea that in the case $T = \{0,1\}$, if there is a no-hole coloring, then such a coloring will come close in span to the optimal (minimal) span. During a visit to RUTCOR by Lanfear, we discussed with him whether this was in fact the case, and indeed, whether there is even always *some* no-hole coloring which comes close to optimal span. Roberts [1993] showed that there can be no-hole colorings which have spans which are very far from the optimal. He also investigated whether or not there is always a no-hole T-coloring which is close to optimal in span, by studying the unit interval graphs which are the simplest case of the r-unit sphere graphs for which T-coloring is especially interesting in practice. He studied the practically important case $T = \{0,1\}$, and showed that if the number of vertices is more than $2\chi(G)-1$, then there is a no-hole T-coloring and all such colorings attain a span which is within one of being optimal. He also showed that if the number of vertices is less than $2\chi(G)-1$, then there is no no-hole T-coloring. Thesis [50] handles the case where the number of

vertices is $2\chi(G)-1$ and generalizes these results to the case $T = \{0,1,\ldots,r\}$. It also studies the graphs which do not have no-hole colorings with $T = \{0,1\}$, and studies $h(G)$, the smallest number of holes left by a T-coloring of G. We find exact values for $h(G)$ for particular graphs and also relate $h(G)$ to the path-covering number and the Hamiltonian completion number of G.

Other variations of graph coloring that have found important practical applications involve the ***k-tuple colorings*** in which every vertex receives a set of k colors and adjacent vertices must receive disjoint sets. These k-tuple colorings have practical applications in problems involving mobile radio frequency assignment, traffic phasing, vehicle maintenance, and task assignment. (See Opsut and Roberts [1981].) Tesman [1989] introduced the idea of combining k-tuple colorings with T-colorings. In thesis [50], we have compiled and advanced results from earlier years on no-hole k-tuple T-colorings, and generalized the results on no-hole T-colorings described in the previous paragraph.

Griggs and Yeh [1992] introduced a variant of T-coloring which they call $L(2,1)$-labelling. Here, we assign nonnegative integers to the vertices of graph G so that adjacent vertices get numbers at least two apart and vertices at distance two get distinct numbers. In thesis [50] we extend and organize work of earlier years on the ***span*** of an $L(2,1)$-labelling, the difference between the largest and smallest label used, and in particular find the span for chordal graphs and for unit interval graphs (which are special cases of chordal graphs).

We have also studied a graph coloring problem that arose in connection with distributed computing. Suppose that the vertices of an n vertex graph are two-colored by white and black in such a way that for each vertex, at least half the vertices in its neighborhood (including itself) are white. What is the fewest number of white vertices there can be? More generally, for a given integer $r > 0$, suppose that for each vertex v and integer $i \leq r$, at least half the vertices in the ball of radius i centered at v are white. Again, as a function of n and r, what is the fewest number of white vertices there can be? These problems are solved in [43].

Considerations of stability have also played a role in our work in the past year. One particular example of research on stability is the work on the size of a stable or independent set in the Clar graph of a benzenoid system in thesis [53], which is discussed in Section 2.3.

In Section 3.2, we discuss our work on iterated roof duality in thesis [52], from which we obtain the exact values of the stability number for the special case of odd K_4-free graphs.

Given a positive integer k, determining whether an arbitrary graph contains a stable set of size at least k is NP-complete. However, there are special classes of graphs for which the ***stability number***, the size of the largest stable set, can be computed in

polynomial time. In some cases, boolean methods can suggest graph theoretical procedures for determining the size of the largest stable set. In paper [19], we derive from boolean methods a transformation which, when it can be applied, builds from a graph G a new graph G' with the same stability number and one less vertex. We also describe a class of graphs for which such a transformation leads to a polynomial algorithm for computing the stability number.

Considerations of graph coloring and stability play a central role in the survey paper [48], which was revised and updated in the past year from an earlier version, and which summarizes a variety of important problems and trends in graph theory, with special emphasis on applications.

2.2. Special Classes of Graphs

Many graph theory problems are extremely difficult when looked at in general, but turn out to be tractable when restricted to a special class of graphs. Hence, research in graph theory has in recent years emphasized the study of rich and interesting special classes of graphs, many of which arise from applications, and for which efficient algorithms can often be found to solve important optimization problems. Our work on special classes of graphs has reflected this point of view.

One of the classes of graphs we have studied is the class of threshold graphs. These graphs were defined by Chvatal and Hammer [1977] and have since been studied intensively (see Golumbic [1980]). Among the applications of threshold graphs are applications to Guttman scaling in measurement theory and to synchronizing parallel processors. A finite graph is called ***k-threshold*** if there is a number h and a linear k-separator w assigning a real number to each vertex so that for any subset S of vertices, the sum of $w(x)$ for x in S is less than h if and only if the subgraph induced by S does not contain a k-clique. ***Threshold graphs*** are 2-threshold graphs. An ***infinite k-threshold graph*** is a graph in which every finite induced subgraph is k-threshold. In paper [20], begun last year, we develop the theory of infinite threshold graphs.

The idea of ***universal graph***, a graph which contains as an induced subgraph all graphs from a given class, goes back to Rado [1964]. This idea is considered fundamental in extremal graph theory and has important applications in computer science and engineering. A graph is called ***T_n-universal*** if it contains every threshold graph with n vertices as an induced subgraph. T_n-universal threshold graphs are of special interest since they are precisely those T_n-universal graphs which do not contain any non-threshold induced subgraph. Minimum T_n-universal graphs, T_n-universal graphs with a minimum number of vertices, are especially interesting because they may contain non-threshold graphs as induced subgraphs. In paper [22], we show that for every $n \geq 3$, there are minimum T_n-universal graphs which are themselves threshold and others which are not. The set of all minimum T_n-universal graphs is described constructively and the proofs

provide a polynomial recursive procedure *which determines for* any threshold graph G with n vertices and for any minimum T_n-universal threshold graph an induced subgraph isomorphic to G. This work is extended in paper [23].

Threshold graphs are also studied in paper [21]. Here we study the Laplacian spectra, the Laplacian polynomials, and the number of spanning trees of threshold graphs. We give formulas for these parameters in terms of so-called composition sequences of threshold graphs and show that the degree sequence of a threshold graph and the sequence of eigenvalues of its Laplacian matrix are "almost the same." Threshold graphs are shown to be uniquely determined by their spectrum and a polynomial time procedure is given for testing whether a given sequence of numbers is the spectrum of a threshold graph.

Related to threshold graphs are ***threshold boolean functions***, functions for which there exists a hyperplane separating their set of true points from their set of false points. In the thesis [52] we present a new efficient algorithm for recognizing threshold boolean functions. We also show a hierarchy of generalizations of regular boolean functions, which are themselves natural generalizations of threshold functions. For any of these functions, if the set of minimal true points is given, then the set of maximal false points can be found in polynomial time. A new way of representing positive boolean functions using disjunctive condensed forms is presented. Several polynomial algorithms whose inputs are disjunctive normal forms (DNF's) are generalized to the case when the inputs are disjunctive condensed forms, which are shorter than DNF's.

An important class of graphs with regard to applications is the class of competition graphs and its variants. A graph G is the ***competition graph*** of a digraph D (often assumed acyclic) if $V(G) = V(D)$ and there is an edge between vertices x and y in G if and only if there is a vertex a of D so that (x,a) and (y,a) are arcs of D. These graphs, introduced by Joel Cohen in 1968, arise in communications over noisy channels (cf. the confusion graphs of Shannon). They also arise in the channel assignment problem mentioned above, which is concerned with coloring a competition graph. They arise in large-scale computer models of complex systems (see e.g., Greenberg, Lundgren, and Maybee [1981]). They arise in the study of food webs in ecology (see e.g., Cohen [1978].) See the surveys by Raychaudhuri and Roberts [1985] and Lundgren [1989]. One of the central problems about competition graphs is the recognition problem. In general, this is NP-complete. However, it can be reduced to the question of computing a parameter called the ***competition number***, which is the smallest number of isolated vertices to add to a graph so that the resulting graph is the competition graph of an acyclic digraph. An important tool in the theory of competition graphs is an old result of Roberts that computes the competition number for connected, triangle-free graphs. We have now succeeded in extending this result to the case of connected graphs with exactly one triangle. (See paper [40].)

Competition graphs have been generalized in various ways. Many of these generalizations have been summarized in the survey by Lundgren [1989]. One of them is the ***p-competition graph***, a graph G arising from a digraph D (often assumed to be acyclic) by taking $V(G) = V(D)$ and an edge between vertices x and y in G if and only if there are vertices $a_1, \ldots, a_p$ in D so that (x,a_i), (y,a_i) are arcs of D for $i = 1, \ldots, p$. Corresponding to competition number is the same idea of ***p-competition number***. In paper [39] we obtain the surprising result that the p-competition number can be smaller than the competition number, and in fact arbitrarily smaller.

Of great interest in the past 30 years has been the class of perfect graphs first introduced by Claude Berge. See Golumbic [1980]. We have studied the important class of perfect graphs called interval graphs, which was defined in Section 1.2. As we noted in Section 1.2, these graphs arise in numerous applications. Specifically, we have studied interval graphs in connection with no-hole T-colorings; see Section 2.1. As noted in Section 1.2, we have also studied unit interval graphs and $n^{\sim}$-graphs. Our work on these graphs was motivated by problems of visual perception involving perceptual judgements of betweenness and proximity that might be relevant to those made by pilots or radar systems.

Let q be a positive integer. A ***partial q-coloring*** of a graph G is a set of q pairwise disjoint stable sets $S_1, \ldots, S_q$. We define $\alpha_q(G)$ to be the maximum of $|\cup S_i|$ over all partial q-colorings. If $V(G)$ is partitioned into cliques $C_1, C_2, \ldots$, the corresponding ***q-norm*** is the sum of the terms $\min\{|C_j|,q\}$. We define $\theta_q(G)$ to be the smallest q-norm over all clique partitions of G. In general, $\alpha_q(G)) \leq \theta_q(G)$. For many graphs, these are equal. If a graph G and all its induced subgraphs have this property, we say that G is ***q-perfect***. For $q = 1$, this reduces to the classical concept of perfect graph. In paper [2], we study the graphs which appear to be q-perfect for some values of q. We show for instance that every balanced graph is q-perfect for all $q \geq 1$ and we obtain a characterization of q-perfect graphs for $q \geq 2$ in terms of a linear programming problem.

An important class of graphs, for instance in connection with electronic circuits and VLSI design, is the class of planar graphs. In paper [36], we study graph planarity and related topics. We describe different results on graphs containing or avoiding subdivisions of some special graphs, and in particular, different refinements of Kuratowski's planarity criterion for 3-connected and quasi 4-connected graphs. Some results on non-separating circuits in a graph are presented. Some more refinements of Kuratowski's theorem and graph planarity criteria in terms of non-separating circuits are given for 3-connected and quasi 4-connected graphs. An ear-like decomposition for quasi 4-connected graphs is described similar to that for 3-connected graphs, and is shown to be a very useful tool for investigating graph planarity and some other problems for quasi 4-connected graphs. Refinements of different kinds are given for Whitney's graph planarity criterion. Some results on Dirac's conjecture and Barnette's conjecture are also

presented.

Special classes of graphs play a central role in the survey paper [48], which, as noted in Section 2.1, was revised and updated in the past year from an earlier version, and which summarizes a variety of important problems and trends in graph theory, with special emphasis on applications.

2.3 Graph Theory and Discrete Optimization

Discrete optimization problems arise in a large variety of vitally important practical scheduling, allocation, planning, and decisionmaking problems. We shall have more to say about discrete optimization in Section 3.1. Here, we note that sometimes important classes of graphs are related to problems in discrete optimization. This idea has entered into several pieces of work under this project.

Networks with weights on the arcs occur in many important Air Force applications. We have been interested in such networks in which the arcs represent activities, each activity has a certain required time, the vertices represent stages, and each stage is required to be reached at the same time. In particular, these kinds of problems arise in transportation networks with travel times on the arcs and where synchronized arrivals are required; in PERT networks of activities where all activities leading to a given stage must end at the same time; and in parallel machines in which the arcs correspond to different processors and where each processor requires all of its input signals to arrive at the same time (pipelining the data flow). The problem is to add idle times for each activity so that synchronization can be achieved at each stage in such a way as to minimize the total idle time. In paper [8], we have presented the first polynomial time solution for the problem of finding the idle time assignment which minimizes the total amount of idle time in a network. We have also shown that the problem of minimizing the maximum idle time can be solved in polynomial time, while the problem of minimizing the number of activities with positive delay from some ideal completion time is NP-complete.

Induced subgraphs with a tree structure arise in a variety of problems in telecommunications and design of reliable networks for communication, transportation, and power. In paper [38], we have initiated the study of the problem of optimally packing in a graph the special tree subgraphs called stars.

Classes of graphs which have particular kinds of orientations have played an important role in graph theory and its applications. From the point of view of moving traffic (both vehicles and information), graphs which have strongly connected orientations are of central importance. It has been known for more than fifty years which graphs have strongly connected orientations. What is interesting to ask is what makes one strongly connected orientation better than another. One can introduce a variety of criteria of efficiency of a strongly connected orientation (see Roberts [1976,1978] for many examples). Unfortunately, for none of the

widely studied criteria is the problem of finding the most efficient strongly connected orientation solvable by an efficient algorithm for a general graph. However, for the graphs which arise in practical traffic routing problems, considerable progress has been made. In a series of papers prepared in earlier years, we have studied the grid graphs consisting of north-south streets and east-west avenues, and found optimal strongly connected orientations under two fundamental criteria. Now, in paper [3], we have started to make considerable progress on the same problem for annular cities.

Matchings are of great importance in a variety of applications, including assignment problems for jobs, tasks, and storage, and specifically in assigning pilots to aircraft. We have been concerned with perfect matchings, with an emphasis on their applications to chemistry. The thesis [53] is concerned with perfect matchings in benzenoid systems. It organizes and extends work we have been doing on this subject in past years. A ***benzenoid system*** is defined to be a finite connected subgraph of the infinite hexagonal lattice which has no nonhexagonal interior faces or cut edges. Such graphs are also of considerable importance in communication networks, specifically in mobile radio communication. We have focused on perfect matchings of such graphs. Benzenoid systems, which are the skeletons of benzenoid hydrocarbons, are important for both chemists and combinatorialists. See Cyvin and Gutman [1988]. Vast experimental data show that many chemical properties of benzenoid hydrocarbons, such as stability and color, can be explained by the topological properties and structure of the corresponding benzenoid systems. We have developed two algorithms to determine a ***Kekulé structure*** (or a perfect matching) of a benzenoid system. One of them has linear computational complexity and the other corrects the peeling algorithm of Gutman and Cyvin.

Also in thesis [53] we present a mixed integer programming model to determine ***Clar formulas*** of benzenoid systems or, in other words, maximum sets of mutually resonant hexagons. It turns out that mixed integer programming and, in practice, linear programming, allows us to solve efficiently this problem even for very large molecules (pericondensed benzenoids with several hundred hexagons). We also develop a method to calculate various upper bounds on the ***Clar number***, i.e., the number of mutually resonant hexagons in a Clar formula.

Also in thesis [53], we prove Cyvin and Gutman's conjecture that a normal benzenoid system (or a benzenoid system without fixed bonds) with h (> 1) hexagons can be constructed from a normal benzenoid system with $h-1$ hexagons. This conjecture is the basis of a sieve method to enumerate all normal benzenoid systems with a number of Kekulé structures less than or equal to a given number.

In Section 1.3, we discuss our use of graph theory in paper [27] to solve a problem of bilinear programming. Specifically, we transform problems of reducing quadratic programs to bilinear programs to problems having to do with maximum bipartite subgraphs and maximum stable sets in an associated graph.

In Section 3.1, we discuss a graph-based algorithm developed in paper [24] for the quadratic time minimization of quasi-acyclic Horn functions.

In Section 3.2, we discuss our use in thesis [52] of the method of iterated roof duality for finding bounds on quadratic pseudo-boolean functions to obtain the exact values of the stability number for odd K_4-free graphs..

2.4. Posets, Matroids, and other Useful Combinatorial Structures

As we said at the beginning of Section 2, combinatorial structures such as matroids, graphs, block designs, and partially ordered sets have a wide variety of applications in practical problems. While our work on combinatorial structures has emphasized graphs (see Sections 2.1, 2.2, and 2.3), we have also investigated a variety of other structures and their applications in this project.

Many combinatorial structures arise in the study of geometry. The ***Borsuk conjecture*** concerns such a structure; it states that every subset of Euclidean d-space of unit diameter can be covered by $d+1$ sets each of diameter strictly less than 1. In the book by Croft, Falconer, and Guy [1991], this conjecture is called "one of the most famous unsolved problems of geometry." We have recently solved this problem by showing that the conjecture is wildly incorrect. The result is written up in paper [32].

Special classes of matrices are often of interest in combinatorics. Let A be an $n \times n$ matrix of real numbers. A is called a ***Z-matrix*** if all its off-diagonal entries are nonpositive and an ***F-matrix*** (a ***Ky Fan N-matrix***) if $A = \alpha I - B$ with $B \geq 0$ and $\lambda < \alpha < \rho(B)$, where I is the identity matrix, $\rho(B)$ is the spectral radius of B, and λ is the maximum of the spectral radii of all principal submatrices of B of order $n-1$. We show in paper [47] that if A is a Z-matrix, then it is an F-matrix if and only if a certain linear complementarity problem has exactly two solutions for any positive q and at most two solutions for any other q.

In paper [46], we investigate ***M-matrices***, real matrices A of the form $sI - B$, where $s > \rho(B)$. We investigate the similarity between positive definite matrices and M-matrices. Some well-known inequalities for positive definite matrices are shown to be true for M-matrices. The main results are analogues of the Minkowski, Bergstrom, and Fan inequalities for the difference of two M-matrices.

Partial orders are a very useful combinatorial structure. In Section 3.3 we discuss our applications of randomization methods in paper [35] to the problem of sorting a partially ordered set by comparisons. This problem arises in a variety of practical problems, described in Section 3.3.

One structure of considerable interest in decisionmaking has been the hypergraph. A hypergraph is called ***intersecting*** if no two edges are disjoint. In the mid-1980's, Füredi and Seymour conjectured that if F is an intersecting hypergraph on n vertices, then there is a set of n pairs of vertices such that each edge of F contains one of the pairs. The conjecture is a stronger version of a special case of the Borsuk conjecture discussed above. The fractional version of the Füredi-Seymour conjecture was proved by Füredi and Seymour and then in a stronger form by Alon and Seymour. We have now shown in paper [33] that the Füredi-Seymour conjecture is false. The counterexample is obtained by random methods.

A fundamental result of Pippenger, based on a well-known theorem of Frankl and Rödl, is that for any k-uniform, D-regular, pseudosimple hypergraph H on n vertices, $\rho(H) \sim n/k$, where $\rho(H)$ is the minimum size of a hypergraph $H' \subset H$ whose edges cover the vertex set of H. In paper [34], we obtain a substantial extension of Pippenger's Theorem which essentially describes how far the assumption of pseudosimplicity may be relaxed.

An important class of problems in the theory of computational complexity involves determining the minimum size of circuits for computing various classes of boolean functions. In paper [30] we consider threshold circuits whose basic element is a threshold gate, a multiple input-single output unit described by a linear function of its inputs, which outputs 1 on a certain input if the value of the linear function exceeds 1. Little has been proven about the limitations of such circuits. In this work, we establish a trade-off between the size and depth of these circuits. In particular, our result provides the first superlinear lower bound for an explicit function (in this case, the parity function) on the size of a bounded depth threshold circuit that computes it.

2.5. Random Discrete Structures and their Applications

An increasing theme in discrete mathematical research is to investigate random discrete structures of various kinds. The reason for the emphasis on random structures is in part because of their connections to probabilistic algorithms and in part because of their relevance in formulating models for applied problems. Moreover, sometimes a probabilistic approach can lead to useful results about inherently non-probabilistic problems. We have worked on a number of problems in this area, some concerning behavior of random objects, and others whose solutions seem likely to require a probabilistic approach.

Counting spanning trees is a fundamental problem in enumerative combinatorics. The conventional approach uses determinants, and explicit formulas have been found for several classes of graphs, undirected and directed. In paper [16], begun last year, we concentrate on certain families of digraphs that evolve naturally from the study of certain longstanding unsolved problems in queuing theory, i.e., involving networks of queues with finite buffers. We

derive a new method for counting the number of trees rooted at any given node, and this in turn yields the steady state queue length probabilities for queues of finite length. To demonstrate the method, we have derived explicit formulas for the number of trees, and hence, the queue length probabilities, for the case of two queues in tandem with smallest buffer size at most 2.

In many practical problems of communications, transportation, power distribution, etc., one is dealing with a network which is vulnerable because some of its components are subject to failure. The theory of *network reliability* has been developed to deal with this problem, and it has taken much of its motivation from the literature of command, control, and communications. (See for instance Bracken [1983].) There is a long literature concerning alternative definitions of network reliability and means of computing the reliability of a network under different models of the random failure of components. (See for instance the monograph by Colbourn [1987] and the volume edited by Hwang, Monma and Roberts [1991].) In paper [37], we have found lower and upper bounds on network reliability using operations on graphs that increase or decrease reliability.

The thesis [41] is also concerned with reliability problems, in particular with reliability optimization. The approaches and results are described in Section 3.5.

Randomness is an important aspect of the theory of sorting of information. Our work on this theory, in paper [35], is described in Section 3.3.

Randomness has also been a central theme in the work in paper [31], described in Section 2.1, in which we solve a problem posed by Erdős, Rubin, and Taylor [1979] by finding an asymptotic result for the list chromatic number of the random graph $G(n,1/2)$.

Random methods are used in paper [33] to obtain a counterexample to a conjecture of Füredi and Seymour about intersecting hypergraphs, as described in Section 2.4.

3. Algorithmic Methods

One of the major changes in discrete mathematics in the 1970's and 1980's has been the strong emphasis on algorithms. We have reflected this emphasis throughout the project by studying algorithms for a variety of discrete problems. We have emphasized several themes which we see as increasingly important and which are described in this section.

3.1 Preprocessing and Decomposition Methods for Discrete Optimization Problems

Discrete optimization problems arise in a large variety of vitally important practical scheduling, allocation, planning, and decisionmaking problems. Discrete optimization problems arise frequently in an unmanageable form. One approach is then to

transform a given problem after some manipulation into a more structured one or a small number of more structured problems, for which good solution methods exist. Our research effort has not given as much emphasis to such preprocessing and decomposition of discrete optimization problems in this past year as in earlier years. However, we have obtained a number of results in this spirit.

In Section 1.2, we described our work in paper [5] on an improvement on the k-Means algorithm for clustering. In our algorithm, we have an important preprocessing step in order to reduce the number of objects, followed by an application of quadratic 0-1 minimization methods on the resulting more tractable problem.

Boolean satisfiability problems are central to combinatorial algorithms, both because they encompass many important combinatorial problems and as the first example of a combinatorial problem which is NP-complete. Let V be a set of n boolean variables and V' the set of boolean complements of these variables. The elements of $L = V \cup V'$ are called *literals*. Given a boolean formula in conjunctive normal form (CNF), the ***satisfiability problem*** consists of finding a satisfying true/false assignment to the variables or in recognizing that no such assignment exists. A common simplification of this problem consists of assigning the obvious values to the "pure" literals, i.e., to those which appear only in complemented or only in uncomplemented form. In paper [7], we present a linear time algorithm for determining a subset F^* of the variables and a true/false assignment to these variables such that if there exists a satisfying assignment at all, then there exists one with these same values on F^*. The set of literals fixed by this algorithm includes properly, and can be substantially larger than, the set of pure literals.

Much work has been done to establish subclasses of satisfiability problems which are solvable in polynomial time. In paper [6] we associate a real-valued index to each instance of satisfiability and show that, in some sense, this index measures how hard the problem is. We give an algorithm for satisfiability which runs in polynomial time on any instance for which the value of this index is below a certain threshold. In contrast, we show that for instances where the index is larger than the threshold the problem is NP-complete.

As above, let V be a set of n boolean variables and V' the set of boolean complements of these variables. A ***partial assignment*** is a subset S of literals so that $S \cap S' = \phi$. A ***Horn formula*** is a boolean formula on these n variables so that for all partial assignments L, $|L \cap V'| \leq 1$. Horn formulae play a prominent role in artificial intelligence and logic programming. Their importance is due to a large extent to the fact that for such expressions the satisfiability problem is linearly solvable.

The class of q-Horn boolean expressions, generalizing quadratic, Horn, and disguised Horn formulae, was introduced in

Boros, Crama, and Hammer [1990], in which it is shown that the satisfiability problem corresponding to such an expression is solvable in time linear in the size of the expression. The recognition of such formulae, however, was based on the solution of a linear programming problem, and had therefore a much higher complexity. In paper [9], a combinatorial algorithm is presented for recognizing q-Horn formulae in linear time. Similar results are described in the thesis [52].

In paper [25], we introduce the concept of a ***Horn function***, a boolean function which admits a representation by a Horn formula. Horn functions arise in a variety of applications, including in particular the analysis of production rule knowledge bases of propositional expert systems. In the paper we observe that the irredundant prime disjunctive normal forms (DNF's) of any Horn function are Horn. We reduce the study of the irredundant prime DNF's of Horn functions to the study of the irredundant prime DNF's of pure Horn functions. This reduction is achieved by proving that every prime irredundant DNF of a Horn function consists of a prime irredundant DNF of its "pure Horn component," and of a "positive restriction" of the function. We provide a constructive characterization of all the positive restrictions, and present an efficient algorithm for decomposing any Horn function into its pure Horn component and its positive restriction. Finally, we reduce in quadratic time the problem of minimizing the number of terms in DNF of a Horn function to the same problem for its pure Horn component.

In paper [24], we formalize the compression of knowledge bases as the problem of boolean function minimization and investigate this problem for the widely used class of propositional Horn clause bases. We use the concept of Horn function and consider the special class of quasi-acyclic Horn functions, which properly includes the two practically significant classes of quadratic and of acyclic functions. We develop a cubic time procedure for recognizing the quasi-acyclicity of a function given by a Horn CNF. A graph-based algorithm is proposed for the quadratic time minimization of quasi-acyclic Horn functions.

In paper [26], we continue the investigation of the problem of optimal compression of propositional Horn production rule knowledge bases. The standard approach to this problem, consisting of the removal of redundant rules from a knowledge base, leads to an "irredundant" but not necessarily optimal knowledge base. We prove here that the number of rules in any irredundant Horn knowledge base involving n propositional variables is at most $n-1$ times the minimum possible number of rules. In order to formalize the optimal compression problem, we define a boolean function of a knowledge base as being the function whose set of true points is the set of models of the knowledge base. In this way, the optimal compression of production rule knowledge bases becomes a problem of boolean function minimization. In this paper we prove that the minimization of Horn functions (i.e., boolean functions associated to Horn knowledge bases) is NP-complete.

Other preprocessing methods have been used in our study of

various types of mathematical programming problems. For instance, indefinite quadratic programs with quadratic constraints can be reduced to bilinear programs with bilinear constraints by duplication of variables, as described in Section 1.3. In paper [27], described in that section, we study such reductions in which the number of additional variables is minimum or the number of complicating variables, i.e., variables to be fixed in order to obtain a linear program in the resulting bilinear program, is minimum.

In bilevel programming as described in Section 1.4, several decisionmakers with different objectives intervene in the decisionmaking process. We discuss there our work in paper [28] in which a new branch-and-bound algorithm for linear bilevel programming is proposed. Necessary optimality conditions expressed in terms of tightness of the follower's constraints are used to fathom or simplify subproblems.

3.2. Approximation

A major theme in discrete mathematics in recent years has been to find methods for approximating solutions to problems and to find exact solutions by successive approximations. The approximation problem has been an important focus of our efforts.

Flow problems on networks belong to the most studied problems of mathematical programming. They have numerous applications in practice since highway, rail, electrical, communication and many other physical networks have widespread use. We have used a network flow based algorithm in thesis [52] to study the NP-hard problem of minimizing ***quadratic pseudo-boolean functions***, i.e., quadratic real-valued polynomials whose variables take only the values 0 and 1. The network flow algorithm finds a lower bound for the minimum. This approach gives the same lower bounds as others (such as the well-known roof duality of Hammer, Hansen, and Simeone [1984]), but provides a faster algorithm to compute the lower bound. We also note how the max-flow approach can also quickly identify the optimal values of a subset of variables and report computational results.

Also in thesis [52], we provide better bounds than roof duality by presenting an approach called iterated roof duality. We show that iterated roof duality applied to a class of quadratic pseudo-boolean functions which are naturally associated to graphs provides the exact values of the stability number for the special case of odd K_4-free graphs.

Roof duality also plays a role in the paper [10]. We consider there the ***weighted maximum 2-satisfiability problem***: given a quadratic formula in CNF, let a positive weight be associated with each clause and find a truth assignment maximizing the total weight of the clauses that are satisfied. This problem is equivalent to the problem of finding the minimum z^* of a quadratic posiform. We describe a polynomial time algorithm for computing a lower bound on z^*. The algorithm consists of a finite sequence of elementary boolean operations of two types: fusions $(x + x' = 1)$ and

exchanges $(x + x'y = y + y'x)$. Our main result is that the bound obtained by this method is equivalent to the roof duality bound, which is known to be computable by linear programming. Furthermore, one can check in polynomial time whether such bound coincides with z^*. If not, one can obtain strictly sharper lower bounds making use of two further elementary boolean operations called condensation and consensus.

In paper [12], we discuss the vertex enumeration problem for polytopes, both in an off-line and on-line manner. We describe this work in detail in Section 3.4. Among its many applications, also described there, is that a solution to the off-line problem allows one to solve the approximation problem of finding all near-optimal solutions to a linear program.

In paper [18], we give a brief and elementary proof of a result of Hoffman [1952] about approximate solutions to systems of linear inequalities. We improve upon the earlier results in several ways. First, we obtain a simple proof which relies only on linear programming duality; second, we obtain geometric and algebraic representations of the bounds that are determined explicitly in terms of the underlying matrix; and finally, our bounds with respect to general norms are sharper than those obtained previously.

3.3. Probability and Algorithms

It has long been known that many algorithms which can be bad in their worst cases are very good in an "average" case. This has led to increased interest in analysis of algorithms over random instances of problems. Here, the inputs are drawn from a known distribution and we seek algorithms with good average case behavior. Recent studies of the average case behavior of the simplex algorithm are important examples of what we have in mind. Probabilistic ideas enter into the development of efficient algorithms in another way as well. Namely, sometimes if we allow a machine to make some random choices, we obtain an algorithm -- a random algorithm -- which is very effective at solving a problem. We have studied the interaction between probability and algorithms in both of these ways.

Partial orders are a very useful combinatorial structure. In a variety of practical decisionmaking problems, one is given data in the form of a partial order and is asked to extend it to a linear order. This occurs for example when individual preferences are expressed and a linear order of alternatives is required. It also occurs in activity networks in project planning, where for some activities x and y, we are told that x must be completed before y can be started; and in designing a glossary when we wish to define each word before it is used. The linear extension problem is also important in many problems of sorting in computer science. In work begun last year, we have made in paper [35] an important breakthrough on the problem of finding the best way to sort a partially ordered set by comparisons, a problem originally considered by Fredman in the 1970's. Sorting trivially requires at least $\log(e(P))$ comparisons for a partially ordered set P with

e(P) linear extensions or possible sorts. Fredman showed in 1976 that $\log(e(P)) + 2n$ comparisons suffice, where $n = |P|$. Kahn and Saks showed in 1984 that it can be done in $O(\log(e(P))$ comparisons, by proving the existence of a "good" comparison, meaning one which more or less splits the extensions. In both cases, finding the desired comparisons seems quite intractable. We have now shown that one can sort a partially ordered set P using $O(\log(e(P))$ comparisons and find the comparisons in deterministic polynomial time. This is a major development, which makes the results useful in practice. The results take a completely new approach to the problem, based on entropy of the comparability graph and convex minimization via the well-known ellipsoid algorithm. The results present a nice example of the usefulness of recent powerful randomized methods for volume computation (Dyer, Frieze, and Kannan [1989], Karzanov and Khachiyan [1991]) based on rapidly mixing Markov chains, and the trick is to allow randomization.

In Section 3.4, we describe our work in paper [4] on on-line algorithms for server problems. Randomized algorithms play a central role.

The ***witness finding problem*** for a family R of subsets of a finite set X is to produce a subset S of X that intersects each member of R. Finding deterministic algorithms for such problems seems to be an important step in eliminating randomness from algorithms. In paper [42] we have developed a solution for the case that X is a product set and R is the set of combinatorial rectangles of relative measure bounded below by a fixed constant.

3.4. On-Line Methods

There is increasing emphasis in practical problems to find solution algorithms which are on-line in the sense that one is forced to make choices at the time data becomes available, rather than after having the entire problem spelled out. A large effort, the U.S. Transcom/DARPA Planning and Scheduling Initiative, is devoted to the development of a highly interactive strategic mobility modelling tool which will allow an Air Force user to get detailed information about many aspects of the Air Force transportation systems and make on-line decisions. The emphasis on such a modelling tool underlines the importance of on-line methods for current Air Force problems.

A general approach to on-line problems is to think of them as sequential decisionmaking problems. There are two points of view: (a) formulate a probabilistic model of the future and minimize the expected cost of future decisions; (b) compare an on-line decision strategy to the optimal off-line algorithm, one that works with complete knowledge of the future. The first is the approach taken for instance in the theory of Markov decision models. We have emphasized the second approach.

In an on-line computational problem, the algorithm determines a sequence of actions in response to a growing sequence of "requests" from the environment, with the goal being to minimize some cost

function. An important measure of quality of an on-line algorithm, which has become standard in the literature, is the competitive ratio, which measures, roughly, the worst-case over all input sequences, of the ratio of the cost incurred by the algorithm to the cost that an off-line algorithm could have paid on that sequence. Much effort has been expended to determine, for a wide range of problems, how small a competitive ratio can be obtained. This computational setting is one in which randomized algorithms can be shown to have a provable advantage over deterministic ones. In paper [4] we have made important progress in determining the extent of this advantage for a class of problems called *server problems*. Here, the algorithm controls a set of k servers, which sit in some metric space. At each time step, the environment makes a request, which is represented by a point in the metric space. The algorithm must serve the request by moving a server to the point (if none is there already). The algorithm is charged a cost equal to the total distance moved. Such problems arise naturally in many of the Air Force server problems arising at MAC, specifically in problems where planes (servers) from k hubs are requested to deliver cargo or passengers to different destinations. We have developed a new analytic tool for analyzing the optimal competitive ratio attainable by a randomized algorithm for server problems. This tool provides a way to decompose a server problem into smaller ones and relate the competitive ratio of the given problem to the competitive ratios of the smaller ones. As a consequence, we have determined upper and lower bounds and useful new algorithms for server problems.

The ***off-line vertex enumeration problem*** for polytopes consists of determining all vertices of a given polytope P. The ***on-line vertex enumeration problem*** consists of determining all vertices of $P \cap H$, where H is a given half-space, assuming vertices of P are known. In paper [12], begun last year, we discuss both of these problems. The off-line problem can be solved by searching the adjacency graph of P, pivoting iteratively from a tableau corresponding to a vertex of P to a tableau corresponding to an adjacent unexplored vertex. Improving on a result of Dyer, we first show that this can be done in $O(mnv)$ time, where m is the number of facets, n is the dimension, and v is the number of vertices of P. Adjacent vertices of a vertex for which a tableau is known can be determined using only two columns of that tableau each time, i.e., by partial pivoting. We discuss and compare several algorithms from the literature that use partial pivoting. We propose a new algorithm in which the number of vertices for which the whole tableau must be built is reduced. This is attained by using hash coding to detect adjacencies between such vertices and their neighbors as well as between pairs of these neighbors. Computational results are reported. Finally we note that the on-line vertex enumeration problem can be stated as an off-line problem after elimination of a variable. This strategy is compared theoretically and/or empirically with recent algorithms for on-line vertex enumeration given in the literature.

3.5. Heuristics

As more and more problems are shown to be difficult, for

instance by proving them to be NP-complete, there is an increasing emphasis on heuristic solutions. Heuristic algorithms are especially important in practice where there are many problems involving hundreds, thousands, even tens of thousands of variables. In such a case, we would like to elaborate a heuristic algorithm capable (in most cases) of very rapidly finding (approximate) solutions to large problems.

Many of the algorithms we have been developing are heuristic in nature. They have been described elsewhere in this report. In this section, we mention just a few additional examples of our work where heuristics play a very central role.

Automated manufacturing systems play a vital role in increasing productivity in manufacturing. The most recent among these systems are characterized by extremely high levels of speed and accuracy. Unfortunately, the complexity of the planning problems associated with these systems usually increases in direct relation to their technological sophistication. We have made important progress on several planning and scheduling problems related to automated manufacturing. In paper [15], we have also successfully addressed several types of tool management problems. A central issue in tool management for flexible manufacturing systems consists in deciding how to sequence the parts to be produced and what tools to allocate to the machines in order to minimize the number of tool setups (which disrupt the production flow). We have developed a "column generation" approach which has allowed us to solve to optimality much larger instances of certain tool-generating problems than those previously handled in the literature.

In Section 2.5 we have discussed some of our work on reliability of networks. Large reliability optimization problems in series-parallel or in complex systems are difficult to solve. A tabu search heuristic is provided in thesis [41] to determine the optimum or near-optimum number of redundant components in each subsystem. For the case of series-parallel systems redundancy optimization can be expressed as a zero-one linear program with generalized upper bounding (GUB) structure. We extend the algorithm of Dantzig and Van Slyke for linear programming with GUB structure to the case of zero-one variables. The resulting algorithm allows us to solve rapidly problems with several hundred variables. In the thesis [41], we also study two-terminal and all-terminal reliability of general networks, with independent probabilities of failure on the arcs, using Boole-Bonferroni inequalities (cf. Prekopa [1988]).

The oil pipeline network problem (closely related to the one-terminal TELPAK problem in telecommunications) consists of determining the layout and diameters of the pipes of a network that connect a given set of offshore platforms and onshore wells to a port. We have designed a specialized implicit enumeration algorithm, extending the one developed for reliability optimization and mentioned in the previous paragraph. This is described in thesis [41]. We also define two new types of valid inequalities exploiting geometric properties of feasible solutions. Finding those of the first type amounts to solving multiple choice knapsack

problems and those of the second type to enumerating spanning trees. Algorithms and implementation for counting and enumerating spanning trees of graphs are included in [41]. A large application is made with simulated data from south Gabon oil field.

Assignment of the modules of a parallel program to processors of a multiple computer system has been studied by Bokhari. He proposed algorithms to solve optimally the following problems: (i) partition chain-structured parallel or pipeline programs over chain-connected systems; (ii) partition multiple chain-structured parallel or pipelined programs over single-host multiple-satellite systems; (iii) partition multiple arbitrarily structured serial programs over single-host multiple-satellite systems; (iv) partition single tree-structured parallel or pipelined programs over single-host multiple identical satellite systems. In thesis [41] we provide algorithms with reduced computational complexity for all four problems.

References

Note: References by number refer to papers listed in the list of publications under the grant.

Boros, E., Crama, Y., and Hammer, P.L., "Polynomial-time Inference of all Valid Implications for Horn and Related Formulae," *Ann. of Math. and Artificial Intell.*, 1 (1990), 21-32.

Boros, E., and Prekopa, A., "Closed Form Two-Sided Bounds for Probabilities that at Least r and Exactly r Out of n Events Occur," *Math.of Oper. Res.*, 14 (1989), 317-342.

Bracken, P., *The Command and Control of Nuclear Forces*, Yale University Press, New Haven, 1983.

Brown, J.I., Kelly, D., Schönheim, J., and Woodrow, R.E., "Graph Coloring Satisfying Restraints" *Discrete Math.*, 80 (1990), 123-143.

Chvatal, V., and Hammer, P. L., "Aggregation of Inequalities in Linear Programming," *Ann. Discrete Math.*, 1 (1977), 145-162.

Cohen, J.E., *Food Webs and Niche Space*, Princeton University Press, Princeton, N.J., 1978.

Colbourn, C.J., *The Combinatorics of Network Reliability*, Oxford University Press, New York, 1987.

Croft, H.T., Falconer, K.J.., and Guy, R.K., *Unsolved Problems in Geometry*, Springer, New York, 1991.

Cyvin, S.J, and Gutman, I., *Kekulé Structures in Benzenoid Hydrocarbons*, Lecture Notes in Chemistry 46, Springer, Berlin, 1988.

Dyer, M.E., Frieze, A.M., and Kannan, R., "A Random Polynomial Time Algorithm for Approximating the Volume of Convex Bodies," *Proc. 21st Annual ACM Symposium on Theory of Computing* (1989), 355-361.

Erdös, P., Rubin, A.L., and Taylor, H., "Choosability in Graphs," *Congressus Numerantium*, 26 (1979), 125-157.

Golumbic, M.C., *Algorithmic Graph Theory and Perfect Graphs*, Academic Press, New York, 1980.

Goodman, N., *The Structure of Appearance*, Harvard University Press, Cambridge, MA, 1951.

Greenberg, H.J., Lundgren, J.R., and Maybee, J.S., "Graph-Theoretic Foundations of Computer-assisted Analysis," in H.J. Greenberg and J.S. Maybee (eds.), *Computer Assisted Analysis and Model Simplification*, Academic Press, New York, 1981.

Griggs, J.R., and Yeh, R., "Labelling Graphs with a Condition at Distance Two," *SIAM J. Discr. Math.*, 5 (1992), 586-595.

Hammer, P.L., Hansen, P., and Simeone, B., "Roof Duality, Complementation and Persistence in 0-1 Optimization," *Math. Programming*, 28 (1984), 121-155.

Hansen, P., Labbé, M., Peeters, P., Thisse, J-F., and Henderson, J.V., "Systems of Cities and Facility Location," in *Fundamentals of Pure and Applied Economics*, Vol. 22, Harwood, Chur, 1987.

Hoffman, A.J., "On Approximate Solutions of Systems of Linear Inequalities," *J. Res. Nat. Bur. Stand.*, 49 (1952), 263-265.

Holzman, R., "An Axiomatic Approach to Location on Networks," *Math. of Oper. Res.*, 15 (1990), 553-563.

Hwang, F., Monma, C., and Roberts, F.S. (eds.), *Reliability of Computer and Communication Networks*, DIMACS Series, Vol. 5, American Mathematical Society and Association for Computing Machinery, Providence, RI, 1991.

Karzanov, A., and Khachiyan, L., "On the Conductance of Order Markov Chains," *Order*, 8 (1991), 7-15.

Lundgren, J.R., "Food Webs, Competition Graphs, Competition-Common Enemy Graphs, and Niche Graphs," in F.S. Roberts (ed.), *Applications of Combinatorics and Graph Theory in the Biological and Social Sciences*, IMA Volumes in Mathematics and its Applications, Vol. 17, Springer-Verlag, New York, 1989, pp. 221-243.

Opsut, R.J., and Roberts, F.S., "On the Fleet Maintenance, Mobile Radio Frequency, Task Assignment, and Traffic Phasing Problems," in G. Chartrand, et al. (eds.), *The Theory and Applications of Graphs*, Wiley, New York, 1981, pp. 479-492.

Prekopa, A., "Boole-Bonferroni Inequalities and Linear Programming," *Oper. Res.*, 36 (1988), 145-162.

Rado, R., "Universal Graphs and Universal Functions," *Acta Arith.*, 9 (1964), 331-340.

Raychaudhuri, A., and Roberts, F.S., "Generalized Competition Graphs and their Applications," in P. Brucker and R. Pauly (eds.), *Methods of Operations Research*, Vol. 49, Anton Hain, Konigstein, West Germany, 1985, 295-311.

Roberts, F.S., *Discrete Mathematical Models, with Applications to Social, Biological, and Environmental Problems*, Prentice-Hall, Englewood Cliffs, NJ, 1976.

Roberts, F.S., *Graph Theory and its Applications to Problems of Society*, CBMS-NSF Monograph #29, SIAM, Philadelphia, PA, 1978.

Roberts, F.S., "On the Mobile Radio Frequency Assignment Problem and the Traffic Light Phasing Problem," *Ann. N.Y. Acad. Sci.*, 319 (1979), 466-483.

Roberts, F.S., *Applied Combinatorics*, Prentice-Hall, Englewood Cliffs, N.J., 1984.

Roberts, F.S., "From Garbage to Rainbows: Generalizations of Graph Coloring and their Applications," in Y. Alavi, G. Chartrand, O.R. Ollermann, and A.J. Schwenk (eds.), *Graph Theory, Combinatorics, and Applications*, Vol. 2, Wiley, New York, 1991, pp. 1031-1052. (a)

Roberts, F.S., "T-colorings of Graphs: Recent Results and Open Problems," *Disc. Math.*, 93 (1991), 229-245. (b)

Roberts, F.S., "No-Hole 2-Distant Colorings," *Math. and Computer Modelling*, 1993, to appear.

Tesman, B., *T-Colorings, List T-Colorings, and Set T-Colorings of Graphs*, Ph.D. Thesis, Department of Mathematics, Rutgers University, New Brunswick, NJ, October 1989.

Vohra, R.V., "An Axiomatic Characterization of Some Locations in Trees," mimeographed, Faculty of Management Sciences, Ohio State University, Columbus, OH, March 1990.

Weber, A., *Über den Standort der Industrien*, Tübingen, Germany, 1909. Translated as *Alfred Weber's Theory of the Location of Industries*, University of Chicago, Chicago, IL, 1929.

RUTCOR

Discrete Methods and their Applications

Grant Number AFOSR 90-0008

List of Publications: October 1, 1991 - September 30, 1992

Note: RRR means RUTCOR Research Report

1. Abeledo, H., *The Stable Matching Problem*, Ph.D. Thesis, RUTCOR, June 1993, to appear.

2. Berge, C., "The q-Perfect Graphs," RRR 23-92, July 1992.

3. Bermond, J-C., and Roberts, F.S., "Strongly Connected Orientations of Annular Cities," in preparation.

4. Blum, A., Karloff, H., Rabani, Y., and Saks, M., "A Decomposition Theorem and Bounds for Randomized Server Problems," *Proc. of 33rd IEEE Symp. on Foundations of Computer Science*, November 1992, 197-207.

5. Boros, E., " A Quadratic 0-1 Minimization Algorithm for Clustering," in preparation.

6. Boros, E., Crama, Y., Hammer, P.L., and Saks, M., "A Complexity Index for Satisfiability Problems," RRR 9-92, March 1992. To appear in *SIAM J. on Computing*.

7. Boros, E., and Hammer, P.L., "A Generalization of the Pure Literal Rule for Satisfiability Problems," RRR 20-92, April 1992.

8. Boros, E., Hammer, P.L., Hartmann, M.E., and Shamir, R., "Balancing Problems in Acyclic Networks," RRR 7-92, March 1992. To appear in *Discrete Appl. Math.*

9. Boros, E., Hammer, P.L., and Sun, X., "Recognition of Q-Horn Formulae in Linear Time," RRR 19-92, April 1992.

10. Bourjolly, J-M., Hammer, P.L., Pulleyblank, W.R., and Simeone, B., "Boolean-Combinatorial Bounding of Maximum 2-Satisfiability," RRR 5-92, January 1992.

11. Chen, P-C., *Vertex Enumeration Methods and Continuous Location Theory*, Ph.D. Thesis, RUTCOR, October 1992.

12. Chen, P-C., Hansen, P., and Jaumard, B., "Partial Pivoting in Vertex Enumeration," RRR 10-92, March 1992.

13. Chen, P-C., Hansen, P., Jaumard, B., and Tuy, H., "Solution of the Multifacility Weber and Conditional Weber Problems by D.-C. Programming," RRR 22-92, June 1992.

14. Chen, P-C., Hansen, P., Jaumard, B., and Tuy, H., "Weber's Problem with Attraction and Repulsion," RRR 13-92, April 1992 (revision of RRR 62-91).

15. Crama, Y., and Oerlemans, A.G., "A Column Generation Approach to Job Grouping for Flexible Manufacturing Systems," RRR 15-92, May 1992.

16. Dean, N., Winkler, P., Kelmans, A.K., Lih, K-W., and Massey, W.A., "The Spanning Tree Enumeration Problem for Digraphs," mimeographed, September 1992.

17. Eeckhoudt, L., and Hansen, P., "Mean-Preserving Changes in Risk with Tail Dominance," RRR 2-92, January 1992.

18. Guler, O., Hoffman, A.J., and Rothblum, U.G., "Approximations to Solutions to Systems of Linear Inequalities," RRR 30-92, September 1992.

19. Hammer, P.L., and Hertz, A., "On a Transformation which Preserves the Stability Number," RRR 69-91, December 1991.

20. Hammer, P.L., and Kelmans, A.K., "Infinite Threshold Graphs," in preparation.

21. Hammer, P.L., and Kelmans, A.K., "Laplacian Spectra and Spanning Trees of Threshold Graphs," RRR 36-92, September 1992.

22. Hammer, P.L., and Kelmans, A.K., "On Universal Threshold Graphs," RRR 14-92, April 1992. To appear in *Discr. Appl. Math.*

23. Hammer, P.L., and Kelmans, A.K., "Universal and Locally Threshold Graphs," in preparation.

24. Hammer, P.L., and Kogan, A., "Horn Function Minimization and Knowledge Compression in Production Rule Bases," RRR 8-92, March 1992.

25. Hammer, P.L., and Kogan, A., "Horn Functions and their DNF's," RRR 6-92, February 1992. To appear in *Info. Proc. Letters*.

26. Hammer, P.L., and Kogan, A., "Optimal Compression of Propositional Horn Knowledge Bases: Complexity and Approximation," RRR 1-93, January 1993.

27. Hansen, P., and Jaumard, B., "Reduction of Indefinite Quadratic Programs to Bilinear Programs," RRR 3-92, January 1992, to appear in *J. Global Optimization*.

28. Hansen, P., Jaumard, B., and Savard, G., "New Branch and Bound Rules for Linear Bilevel Programming," RRR 12-92, to appear in *SIAM J. Sci. and Stat. Computing*.

29. Hansen, P., and Roberts, F.S., "An Impossibility Result in Axiomatic Location Theory," RRR 1-92, January 1992; revision, mimeographed, June 1992.

30. Impagliazzo, R., Paturi, R., and Saks, M., "Size-depth Trade-offs for Threshold Circuits," accepted for 1993 *Symposium on Theory of Computing*.

31. Kahn, J., "Asymptotically Good List Colorings," in preparation.

32. Kahn, J., and Kalai, G., "A Counterexample to Borsuk's Conjecture," RRR 42-92, December 1992, to appear in *Bull. Amer. Math. Soc.*

33. Kahn, J., and Kalai, G., "A Problem of Füredi and Seymour on Covering Intersecting Families by Pairs," mimeographed.

34. Kahn, J., and Kayll, M., "On a Theorem of Frankl and Rödl II," in preparation.

35. Kahn, J., and Kim, J.H., "Entropy and Sorting," to appear in J. *ACM*.

36. Kelmans, A.K., "Graph Planarity and Related Topics," RRR 28-92, September 1992. To appear in N. Robertson and P. Seymour (eds.), *Proc. Seattle Conference on Graph Minors*.

37. Kelmans, A.K., "Lower and Upper Bounds for Network Reliability," in preparation.

38. Kelmans, A.K., "Optimal Packing of Induced Stars in a Graph," in preparation.

39. Kim, S-R., McKee, T., McMorris, F.R., and Roberts, F.S., "p-Competition Numbers," mimeographed. To appear in *Discr. Appl. Math.*

40. Kim, S-R., and Roberts, F.S., "Competition Numbers of Graphs with One Triangle," in preparation.

41. Lih, K-W., *Applications of Zero-One Linear Programming with Multiple-Choice Constraints*, Ph.D. Thesis, RUTCOR, January 1993.

42. Linial, N., Luby, M., Saks, M., and Zuckerman, D., "Efficient Construction of a Small Hitting Set for Combinatorial Rectangles in High Dimension," *Proc. 25th ACM Symposium on Theory of Computing*, to appear.

43. Linial, N., Peleg, D., Rabinovitch, Y., and Saks, M., "On the Value of Neighborhood Polling," in preparation.

44. Mahadev, N.V.R., and Roberts, F.S., "Amenable Colorings," RRR 21-92, May 1992.

45. McMorris, F.R., and Roberts, F.S., "Medians in Trees," in preparation.

46. Miao, J., "Analog of the Minkowski Inequality for the Difference of Two M-matrices," RRR 35-92, September 1992.

47. Miao, J., "Ky Fan's N-matrices and Linear Complementarity Problems," RRR 34-92, September 1992.

48. Roberts, F.S., "New Directions in Graph Theory," *Discrete Appl. Math.*, to appear.

49. Roberts, F.S., "On the Median Procedure," in B. Bouchon-Meunier, L. Valverde, and R. Yager (eds.), *Intelligent Systems with Uncertainty*, Elsevier, 1993, to appear.

50. Sakai, D., *Generalized Graph Colorings and Unit Interval* Graphs, Ph.D. thesis, RUTCOR, September 1992.

51. Sakai, D., "On Properties of Unit Interval Graphs with a Perceptual Motivation," mimeographed.

52. Sun, X., *Combinatorial Algorithms for Boolean and Pseudo-Boolean Functions*, Ph.D. Thesis, RUTCOR, October 1992.

53. Zheng, M., *Perfect Matchings in Benzenoid Systems*, Ph.D. Thesis, RUTCOR, October 1992.

RUTCOR

Discrete Methods and their Applications

Grant Number AFOSR 90-0008

Lectures Delivered and Miscellaneous Honors

October 1, 1991 - September 30, 1992

Miscellaneous Honors

Fred Roberts' work on traffic flow was written up in a featured article in the *Chronicle of Higher Education* in January 1992. A copy of the article is attached.

Lectures Delivered

Peter L. Hammer

"On a Polynomially Solvable Class of Satisfiability Problems," at ORSA/TIMS joint National Meeting, Anaheim, CA, November 1991.

"Network Flows and Roof Duality for Quadratic 0-1 Programming," at ORSA/TIMS joint National Meeting, Anaheim, CA, November 1991.

"Reducibility of Set Covering to a Knapsack Problem," plenary talk at Computer Science and Operations Research Conference, Williamsburg, Virginia, January 1992.

"A Complexity Index for Satisfiability Problems," plenary talk at Conference on Integer Programming and Combinatorial Optimization, Carnegie-Mellon University, Pittsburgh, PA, May 1992.

"Max-cut via Cycle Packings," plenary talk at EURO XII, Helsinki, Finland June 1992.

"Satisfiability of Quadratic Horn Forms," plenary talk at International Conference on Graphs and Optimization," Grimentz, Switzerland, August 1992.

"Boolean Methods in Discrete Optimization," plenary talk at First Russian-Italian Conference on Methdods and Applications of Mathematical Programming, Italy, September 1992.

Fred S. Roberts

"Elementary, Sub-Fibonacci, Regular, Van Lier, and other Interesting Sequences," plenary talk at Sixth Clemson Conference on Discrete Mathematics, Clemson, South Carolina, October 1991.

"The One Way Street Problem," Pi Mu Epsilon Talk, Seton Hall University, South Orange, NJ, November 1991.

"Mathematics, Traffic Flow, and the Environment," invited presentation at Joint Policy Board in Mathematics Press Briefing on Mathematics and the Environment, American Mathematical Society National Meeting, Baltimore, MD, January 1992.

"An Impossibility Result in Axiomatic Location Theory," plenary talk at European Chapter on Combinatorial Optimization, Annual Meeting, Graz, Austria, April 1992.

Four-lecture series on "Applications of Discrete Mathematics" at William Paterson College, Wayne, NJ, March-April 1992.

"Consensus Functions and Patterns in Molecular Sequences," plenary talk at Fourth Stony Brook Biomathematics Conference, Stony Brook, NY, May 1992.

"An Impossibility Result in Axiomatic Location Theory," plenary talk at International Conference on Graph Theory and its Applications, Kalamazoo, MI, June 1992.

Five lectures on Graph Theory and its Applications to DIMACS Leadership Workshop on Discrete Mathematics, New Brunswick, NJ, July 1992.

"On the Median Procedure," invited talk at 4th International Conference on Information Processing and Management of Uncertainty, Palma de Mallorca, Spain, July 1992.

"On the Meaningfulness of Ordinal Comparisons for General Order Relational Systems," plenary talk at European Mathematical Psychology Group, Annual Meeting, Brussels, Belgium, July 1992.

Four invited plenary talks on Measurement Theory, its Applications, Meaningless Statements, and Dimensional Analysis at Conference on Mathematical Systems Underlying Axiomatic Measurement Theories, Irvine, CA, July 1992.

<u>Endre Boros</u>

"On a Polynomially Solvable Class of SAT Problems," invited talk at ORSA/TIMS Joint National Meeting, Anaheim, CA, November 1991.

"A Complexity Index for SAT," plenary talk at Second International Symposium on Artificial Intelligence and Mathematics, Ft. Lauderdale, FL, January 1992.

"Reducibility of Set-Covering to a Knapsack Problem," invited talk at the ORSA Computer Science Technical Section Conference, Williamsburg, VA, January 1992.

"A Complexity Index for SAT Problems," presentation at the 2nd Conference on Integer Programming and Combinatorial Optimization, Carnegie-Mellon University, Pittsburgh, PA, May 1992.

Pey-chun Chen

"Vertex Enumeration with Hash Coding and the Neighborhood Problem," at ORSA/TIMS joint National Meeting, Anaheim, CA, November 1991.

"Generalized Weber Problems and D.C. Programming, at ORSA/TIMS joint National Meeting, Orlando, FL, April 1992.

"Vertex Enumeration Methods and Continuous Location Theory," GERAD, Ecole des Hautes Etudes Commerciales, Montreal, Canada, July 1992.

Pierre Hansen

"On Weber's Problem," plenary lecture at XVIth SOR Meeting, Trier, Germany, October 1991.

"Partial Pivoting in Vertex Enumeration," plenary talk at Second Workshop on Integer Programming and Combinatorial Optimization, Carnegie-Mellon University, Pittsburgh, PA, May 1992.

"Clustering Algorithms," ONR Workshop on Discrete Structures in Classification, Herndon, Virginia, May 1992.

"New Branch and Bound Rules for Linear Bilevel Programming," plenary talk at International Conference on Graphs and Optimization, Grimentz, Switzerland, August 1992.

Jeff Kahn

"A Problem of Erdős and Lovasz and other Stuff," colloquium talk, Rutgers University, New Brunswick, NJ, February 1992.

"Asymptotics of Packing, Covering and Coloring Problems," seminar talk, University of Michigan, Ann Arbor, April 1992.

"Asymptotics of Packing, Covering and Coloring Problems," colloquium talk, Carnegie-Mellon University, Pittsburgh, PA, April 1992.

"A Semirandom Method for Hypergraph Problems," 4 hours of lectures, Hebrew University, Jerusalem, June 1992.

"Geometric Aspects of Sorting Partially Ordered Sets," Technion, Haifa, June 1992.

"Singularity Probabilities for Random {+/-1}-matrices," Technion, Haifa, June 1992.

"Asymptotically Good List Colorings," Technion, Haifa, June 1992.

"Asymptotically Good List Colorings," invited talk in special session on probabilistic combinatorics at American Mathematical

Society/London Mathematical Society meeting, Cambridge, England, June 1992.

"A Counterexample to Borsuk's Conjecture," invited talk in special session on discrete geometry and convexity at American Mathematical Society/London Mathematical Society meeting, Cambridge, England, July 1992.

Alexander Kelmans

"Packing in a Graph Subgraphs of Special Type," at Southeastern International Conference on Combinatorics, Graph Theory, and Computing, Boca Raton, FL, February 1992.

"A Generalization of the Edmonds-Gallai Theorem on Graphs," invited seminar talk, MIT, Cambridge, MA, March 1992.

"Reliable Networks from Non-reliable Elements," invited seminar talk, Boston University, Boston, MA, March 1992.

"Non-Hamiltonian Graphs and Barnette's Conjecture," invited seminar talk, McMaster University, Hamilton, Ontario, Canada, May 1992.

"Some Optimal Packing Problems for a Graph and a New Class of Matroids," invited seminar talk, University of Waterloo, Waterloo, Ontario, Canada, May 1992.

"Random Graphs and Network Reliability," invited seminar talk, York University, Toronto, Ontario, Canada, May 1992.

"Tutte's Conjecture on Bipartite, Cubic, and 3-Connected Graphs," invited seminar talk, University of Waterloo, Waterloo, Ontario, Canada, May 1992.

"Coding of Spanning Trees of the Extended Graphs," invited seminar talk, MIT, Cambridge, MA, May 1992.

"Coding of Spanning Trees of the Extended Graphs," invited seminar talk, Northeastern University, Boston, MA, May 1992.

"Universal Threshold Graphs," Advanced Research Institute in Discrete Applied Mathematics, RUTCOR, June 1992.

"Recent Results on Planarity," invited talked at Seventh International Conference on Graph Theory, Combinatorics, Algorithms, and Applications, June 1992.

"Graph Planarity and Related Topics," plenary talk, Conference on Graph Minors, Seattle, Washington, June-July 1992.

Alexander Kogan

"Logic Minimization," plenary talk at conference on Computer Science

and Operations Research: New Developments in their Interfaces, Williamsburg, VA, January 1992.

"Horn Functions and their DNF's," invited plenary talk at DIMACS Forum, Princeton, NJ, April 1992.

"Horn Function Minimization and Knowledge Compressin in Production Rule Bases," at Seventh Advanced Research Institute in Discrete Applied Mathematics, RUTCOR, Rutgers University, New Brunswick, NJ, June 1992.

Keh-wei Lih

"Solving Mixed-Integer Programming Problems with GUB Constraints," ORSA/TIMS joint National Meeting, Anaheim, CA, November 1991.

"A Mathematical Approach to Oil Pipeline Design," ORSA/TIMS joint National Meeting, Orlando, FL, April 1992.

Uriel Rothblum

"Approximations to Solutions of Linear Inequalities," SIAM meeting, Minneapolis, MN, October 1991.

"Formulation of Linear Problems and Solution by a Universal Machine," invited talk at Matrix Workshop, Institute for Mathematics and its Applications, Minneapolis, MN, November 1991.

"The Optimality of Clustering and Monotone Optimal Assemblies," invited talk, Workshop on Discrete Structures in Classification, Herndon, VA, May 1992.

Denise Sakai

"Unit Interval Graphs, $n^{\sim}$-graphs and Uniform n-Extensions," 23rd Southeastern International Confrence on Combinatorics, Graph Theory and Computing, Boca Raton, FL, February 1992.

"Generalized Graph Colorings," seminar, Seton Hall University, South Orange, NJ, February 1992.

"Generalized Graph Colorings," seminar, City College of Staten Island, Staten Island, NY, February 1992.

"Generalized Graph Colorings," Naval Postgraduate School, Monterey, CA, February 1992.

"Generalized Graph Colorings and the Channel Assignment Problem," seminar, Marist College, Poughkeepsie, NY, March 1992.

"Generalized Graph Colorings," seminar, Montclair State College,

Upper Montclair, NJ, March 1992.

"Generalized Graph Colorings and Intersection Assignments," seminar, University of Chicago, Chicago, IL, March 1992.

Maolin Zheng

"On Clar Number and Clar Formula," Seventh Advanced Research Institute in Discrete Applied Mathematics, RUTCOR, June 1992.

"Perfect Matchings in Benzenoid Systems," GERAD, H.E.C., Montreal, September 1992.

Participants in
RUTCOR Project on "Discrete Methods and their Applications"

October 1, 1991 - September 30, 1992

Faculty

Peter Hammer (Principal Investigator)

Fred Roberts (Principal Investigator)

Endre Boros

Vasek Chvatal

Pierre Hansen

Jeff Kahn

Uriel Rothblum

Michael Saks

Postdoctoral Fellows

Alexander Kelmans

Alexander Kogan

Graduate Students

Tamas Badics

Ondrej Cepek

Pey-Chun Chen

Oya Ekin

Wenzhong Li

Keh-Wei Lih

Jianmin Long

Jianmin Miao

Dale Peterson

Denise Sakai

Li Sheng